

INTERIM JOINT MATRICULATION BOARD
AHMADU BELLO UNIVERSITY
ZARIA



INTERIM JOINT MATRICULATION BOARD EXAMINATION 2016

SUBJECT: 'A' LEVEL MATHEMATICS PAPER II
DATE SCHEDULED: TUESDAY 16TH FEBRUARY, 2016
TIME ALLOWED: TWO HOURS (2 HRS)

Instructions:

- (i) Unless otherwise restricted, the use of mathematical tables is PERMITTED.
- (ii) Use of SCIENTIFIC calculator is ALLOWED.
- (iii) Marks for each question are indicated at the end.
- (iv) Do not spend more than HALF (1/2) HOUR on section A.
- (v) Attempt ALL questions in section A; and FOUR (4) questions from other sections, choosing at least ONE (1) question from each of sections B and C.

SECTION A (20%)

1. Differentiate $\cos^{-1}\left(\frac{1-x}{1+x}\right)$. You may simplify your answer. [04marks]
2. Evaluate $\lim_{x \rightarrow 9} \left(\frac{x-9}{\sqrt{x}-3}\right)$. [04marks]
3. Find the values of β such that $\beta\mathbf{i} - 2\mathbf{j} - \beta\mathbf{k}$ is perpendicular to $\beta\mathbf{i} + 4\mathbf{j} - \beta\mathbf{k}$. [04marks]
4. Solve the equation $\frac{dy}{dt} = t^2(1+y)$. [04marks]
5. Evaluate $\int_0^2 \frac{7x}{10+x^2} dx$. [04marks]

SECTION B: CALCULUS

6. (a). Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at (1,1) if $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. [10marks]
- (b). If $y = e^{5x} \sin 4x$, show that $y'' - 10y' + 41y = 0$. [10marks]
7. (a). Differentiate from first principle $y = 100 + 3x - 5x^3$. [10marks]

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(b). Find $\int \frac{(x^2 + 1)dx}{x(x-1)(x+1)}$. [10marks]

8. (a). If $y = \sinh(k \sin^{-1} x)$, show that $(1 - x^2)y'' - xy' - k^2y = 0$. [06marks]

(b). Using integration by parts, show that $nI_n = \sin x \cos^{n-1} x + (n-1)I_{n-2}$, given that

$I_n = \int \cos^n x dx$. Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x dx$. [14marks]

SECTION C: DIFFERENTIAL EQUATIONS AND VECTORS

9. (a). Solve the equation $(2x - y)dx - ydy = 0$. [12marks]

(b). The vertices of ΔABC are represented by the vectors $2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, respectively. Show that ΔABC is an isosceles triangle and obtain its area. [08marks]

10. (a). Calculate the unit vector in direction of the sum of the vectors $\mathbf{a} = 3\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and

$\mathbf{b} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$. Hence obtain the angle between it and vectors \mathbf{a} . [10marks]

(b). Solve the equation $y \frac{dy}{dx} = xe^{x^2 - 3y^2}$. [10marks]

11. (a). Solve the equation $(x + 2) \frac{dy}{dx} - 2y = (x + 2)^{-1}$. [10marks]

(b). Given that $\mathbf{p} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{q} = \mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$, find the projection of the vector $\mathbf{p} - 2\mathbf{q}$ on $2\mathbf{p} + \mathbf{q}$. [10marks]